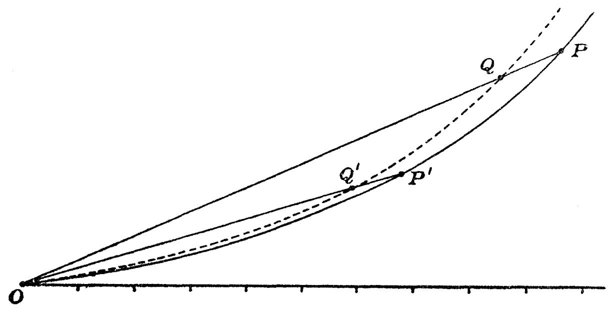
The human brain shows signs of having expanded more decisively in some parts than in others, yet that expansion, if we except the visual and olfactory areas, has been general in kind.

By super-imposing on our cerebral plans drawn from naked-eye inspection others giving the results of histological examination, this as a preliminary to the final localisation of function by the physiologist and workers in other departments, all existing doubt on various homologies may be removed.

"Exterior Ballistics.—'Error of the Day,' and other Corrections to Naval Range-tables." By Professor GEORGE FORBES, F.R.S. Received December 19, 1904,—Read January 26, 1905.

One of the most common problems that meet artillerists is that of correcting for retardation caused by air-resistance, this being proportional to the air-density. The published range-tables, as calculated for any type of gun with given weight and form of shot, and given charge of powder, are based on trials with different elevations when the range and time of flight are measured directly. The range-tables are calculated from the experiments by making corrections for muzzle velocity and for air density. The problem now before us is to find a simple rule for deducing from the published range-table another range-table with a different air-density.



Abscissæ represent ranges.
Ordinates represent elevations.
PP' range table elevations at normal air density.
QQ' „ with air density increased 10 per cent.
$QP = \frac{1}{11} OP$, $Q'P' = \frac{1}{11} OP'$.

The solution of this problem, for flat trajectories at least, is extremely neat when stated geometrically. Draw a curve in which abscissæ are ranges and ordinates are the elevations of the range-table.

To draw a similar curve for air-density increased in the ratio $m : 1$ proceed as follows :—From the origin of co-ordinates O draw straight lines to points P, P′, etc., on the range-table curve. In these lines take points Q, Q′, etc., so that $OP/OQ = OP'/OQ' = \text{etc.} = m$. Then the locus of Q is the new curve.

Stated verbally the rule is :—To make a range-table for x per cent. increase of air-density (or x per cent. increase of retardation due to diameter and weight of shot), diminish each elevation and corresponding range by x per cent. The elevations thus found are correct for the ranges thus found with the new air-density. The time of flight is diminished by x per cent. The remaining velocity found in the range-table is the same for the new range-table at the altered range.

Adopting the usual notation for exterior ballistics—

X is the range in feet; V is the muzzle velocity.
$C = w/nd^2$.
w = weight of projectile; d = diameter of projectile.
n = a constant depending on form of shot and air-density.
v_1 = remaining velocity.
v_0 = velocity at the vertex of trajectory.
t = time of flight; ϕ = elevation.

Certain functions of velocity have been calculated and tabulated. These are S (v), T (v), D (v); and the following are three well-known equations :—

$$S(v_1) = S(V) - \frac{X}{C} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1).$$

From this we find v_1 from the tables;

$$t = C\{T(V) - T(v_1)\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2).$$

Taking T (V), and T (v_1), from the tables, we know t.

Now it is always assumed that for elevations up to 15° the vertex of trajectory is reached in half of the time of flight. Hence

$$T(v_0) = T(V) - \frac{\frac{1}{2}t}{C},$$

whence we find v_0 from the tables, which also give us the values D (V) and D (v_0) in the third equation

$$\phi = C\{D(V) - D(v_0)\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3).$$

If the air-density be now increased in the ratio m to 1, and we use letters with accents for the new conditions,

$$C' = \frac{w}{mnd^2} = \frac{C}{m}.$$

Let us find the elevation for a range X′ where

$$X' = X/m. \qquad \text{Then } X'/C' = X/C.$$

Thus in (1) $X'/C' = X/C$, and V is unchanged. Therefore v_1 remains unchanged, or $v'_1 = v_1$. And in (2) $t'/t = c'/c = 1/m$; or $t' = t/m$.

Also v_0' is found from the equation

$$T(v'_0) = T(V) - \frac{\frac{1}{2}t'}{C'} = T(V) - \frac{\frac{1}{2}t}{C} = T(v_0).$$

Therefore $$v'_0 = v_0.$$

Again, in (3)

$$\phi' = C'\{D(V) - D(v'_0)\} = C'\{D(V) - D(v_0)\} = \frac{C'}{C} \cdot \phi = \frac{\phi}{m}.$$

It appears then that if the air-density be increased m-fold and the range diminished m-fold, the elevation and time of flight must be diminished m-fold, but the remaining velocity is the same.

The above is founded on Niven's formulæ, but those of Siacci and Mayevski lead to the same result.

Another important correction is that of muzzle velocity, which depends upon age of gun and on temperature and nature of explosive. If there were no air-resistance the ordinary formulæ tell us that the elevation for any range varies inversely as the square of the muzzle velocity. I find empirically, by comparison of range-tables based on experiments with different muzzle velocities, that the same law holds over a very considerable range of muzzle velocities up to elevations of 10° at least. This has been tested with the Naval guns of 13·5 inch, 12 inch, 9·2 inch, and 6 inch diameters.

We can now subject these two laws (air-resistance and muzzle velocity) to a very severe test. Take the Naval range-table for 12-inch B.L. Gun, Mark IX, with an 850 lb. shot and muzzle velocity 2433 feet per second and from it calculate (by the two laws above) a range-table for the 6-inch Q.F. gun, Marks III, IV, and VI, with a 100 lb. shot and muzzle velocity 1960 feet per second.

To do this, note that the ratio of

$$\frac{\text{Diameter}^2}{\text{Weight}} \text{ is } \frac{12^2}{6^2} \times \frac{100}{850} = \frac{1}{0 \cdot 4706}.$$

Hence we must multiply the ranges and elevations of the 12-inch gun by 0·4706. We must further multiply the new elevations by 1·540, the ratio of the squares of muzzle velocities. The following table shows the results, and a comparison with the data for the 6-inch gun from the Addenda to Naval Range-Tables, and the differences :—

From range-tables, 12-inch gun. M.V. = 2433 f.s.		Calculated, 6-inch gun. M.V. = 1960 f.s.		From addenda elevation for new range.	Difference.
Range.	Elevation.	New range.	Elevation.	Elevation.	
yards.	° ′	yards.	° ′	° ′	′
2000	1 1	941	0 44	0 45	−1
4000	2 10	1882	1 34	1 38	−4
6000	3 37	2824	2 37	2 39	−2
8000	5 26	3765	3 56	3 54	+2
10000	7 33	4706	5 28	5 24	+4

It need hardly be said that the agreement is sufficient for all practical purposes.

An additional fact, of considerable practical importance, is that there is a very simple approximation to the true correction for elevation due to air-density, obtained by taking it as being a constant multiplied by the product of elevation and range, and percentage variation from the normal air-density. The constant varies with the type of gun and with full, or half, charge; but remains the same for all ranges. Thus with the 6-inch gun, Mark VII, the correction at full charge is $\frac{\text{range} \times \text{elevation}}{1,000,000}$, and at half charge it is $\frac{\text{range} \times \text{elevation}}{1,250,000}$. Curiously enough the same formula, with a different constant, gives the correction, at all ranges, to the elevation, due to rate of change of distance during the time of flight of the shot.

These empirical laws, combined with the muzzle velocity law already mentioned, have led to the design of a most simple arrangement of gun-sights corrected for the individual error of the gun due to wear, and for the error of the day due to temperatures of cordite and of the air, and to barometric pressure, which can be worked by the act of using the range-finder, and automatically corrects for time of flight with changing range.